

Trading Aggregated Flex-Offers via Flexible Orders

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Abstract

Flexibility of small loads, in particular from Electric Vehicles (EVs), has recently attracted a lot of interest due to their possibility of participating in the energy market and the new commercial potentials. Different from existing works, the aggregation techniques proposed in this paper produce flexible aggregated loads from EVs taking into account technical market requirements. The produced aggregated flexible loads fulfill the energy market requirements. They can be further transformed into the so-called flexible orders and be traded in the day-ahead market by a Balance Responsible Party (BRP). As a result, the BRP achieves more than 27% cost reduction in energy purchase based on 2016 real electricity prices.

1 Introduction

The integration of EVs into the Smart Grid reveals new business opportunities by exploiting their inherent flexibility [1, 2]. A market actor that controls the charging rate and time of a portfolio of EVs could acquire financial gain from energy arbitrage [3, 4]. The energy required to charge (and/or discharge) the EVs can be traded through bids in day-ahead and/or regulation market at a minimum cost [5]. Numerous of research studies focus on trading the required energy to charge EVs taking into account different parameters.

An optimization charging approach of EVs that activates the participation in both day-ahead and regulation markets is proposed in [6]. Scheduling techniques of EV charging that aim to the maximization of the market actor's profit and take into account electricity price uncertainty are suggested in [7] and [8]. A risk-based scheduling framework for charging EVs is also proposed in [9]. The suggested algorithm is based on day-ahead prices and takes into account driving activity uncertainties in order to minimize the charging cost of the EVs. Similarly, a day-ahead optimization technique for scheduling EVs considering the impact on the day-ahead prices is suggested in [10]. Both optimization and heuristic techniques for optimal charging of EVs aiming to the maximization of the revenue by utilizing energy storage are proposed in [11].

The main characteristic of the research tackling the energy trading of flexible EV loads is the output of the proposed techniques, i.e., an aggregated scheduled load. Unlike other works, we introduce 3 aggregation techniques that produce *flexible* aggregated loads that can be traded in the market. As a result, the market, not the market actor, schedules the loads from the EVs as part of the trading process, minimizing the uncertainty of bidding. For instance, instead of placing a bid to purchase 30MW in hour 3, the market actor places a bid to purchase 30MW in any hour between hour 1 and 5. The market determines the activation time of the bid. In many cases the technical trading details of the market are omitted and the realization of the suggested techniques becomes very difficult. For instance, the proposed scheduling technique in [12] offers less than 200kW in less than an hour in the regulation power market where the minimum bid is 10MW, in full hours [4]. The bidding strategies proposed in [13] and the high power fluctuations of the scheduling outputs in [6] and in [5], would require single hour independent bids [14] that might not fulfill the energy requirements. A conservative bidding approach for the bidding strategy proposed in [15] covers less than 50% of the energy needed to charge the EVs. On the contrary, in our work we use real technical market requirements derived from a specific order (bid) type, namely, *flexible order* [14], as objectives of our proposed techniques.

Contributions. First, we describe both the so-called flex-offer (FO) model, which captures the flexibility of the EVs, and a realistic market framework where the flexibility is traded. Second, we investigate the market-based FO aggregation problem and its complexity. Third, we introduce 3 heuristic algorithms that take into account real market requirements and produce flexible aggregated FOs that can be then traded through flexible orders in the market. Finally, we compare our proposed techniques with 2 base-line approaches and evaluate both the technical and the financial aspect of their results based on real market prices. We show that our proposed techniques achieve more than 20% cost reduction on average in the purchased energy required to charge from 5K to 40K EVs.

The paper is organized as follows: we introduce the preliminary definitions in Section 2 and we present the problem formulation of market-based aggregation in Section 3. In Section 4, we propose 3 heuristic market-based aggregation techniques and we experimentally evaluate them in Section 5. We conclude the paper in Section 6.

2 Preliminaries

In this section, we describe the EV model that can be used to trade flexibility and the market framework used for trading.

2.1 Electric vehicle model

We consider the energy used to charge EVs to be appropriate for flexible energy trading. The reason is that the lithium-ion batteries of EVs are ample power demand devices and their charge can be time shifted when the EVs are plugged-in for more hours than needed for charging. We consider EVs that can be continuously charged with a power-constant voltage (CP-CV) option [16] and their charge is taking place in the range of 20% to 90% state of charge (SOC) so that the battery life is preserved [17].

In our work, because we take into account time shifted loads, we use the flex-offer (FO) concept [18] to represent the charging of a flexible EV. Thus, we define an FO f to be a tuple $f = (T(f), P(f))$ where $T(f)$ is the start charging flexibility interval and $P(f)$ is the power profile. $T(f) = [t_{es}, t_{ls}]$ where t_{es} and t_{ls} are the *earliest start charging time* and *latest start charging time*, respectively. We define time flexibility (tf) to be the difference between t_{ls} and t_{es} . The *power profile* is a sequence of ($m \in \mathbb{N}_{>0}$) consecutive slices, $P(f) = \langle s^{(1)}, \dots, s^{(m)} \rangle$ where a *slice* $s^{(i)}$ has a power value p measured in kW. The duration of slices is 1 hour.

For instance, an EV is plugged in at a house between 1 and 8 a.m. The EV continuously utilizes 3.7kW for 3.3 hours to be charged. However, energy trading is performed per hour and we also use hourly resolution to model the EVs charging. To respect the hourly granularity, we equally distribute the sum of the energy needed during the first and the last regular charging hours and we reduce power fluctuations in the model. Therefore, we assume that the EV consumes 2.4kWh both during the first and the last charging hours and 3.7kWh during the hours in-between. The EV can be modeled by an FO $f = ([1, 4], \langle 2.4, 3.7, 3.7, 2.4 \rangle)$, see Figure 1a. Next, we describe the market framework where such FOs shall be traded.

2.2 Market framework

The Nordic/Baltic market for electrical energy named Nord Pool is considered in our work. Nord Pool is one of the most mature energy markets [19] and Europe's leading power market [14]. It consists of the day-ahead (Elspot) and intra-day markets. We focus on Elspot because it has one of the largest turnovers in the Nordic system and it also supports flexible energy trading [4]. Trading in Elspot occurs daily through orders (bids). The orders specify the energy amount a BRP desires to buy/sell and the price the BRP is willing to pay/be paid for the corresponding energy. Since 2016, Elspot supports *flexible orders* [14].

When a BRP places a flexible order in Elspot, it states the *name*, the *time interval*, the *price limit*, the *volume*, and the *duration* of the order. The time unit is one hour and volume is expressed in MW. The duration expresses the number of hours during which the order can be activated. The time interval must exceed the duration by at least one hour and expresses the potential activation times of the order. A BRP can place 5 flexible orders during a trading day.

Hypothetically, a BRP could purchase the energy needed to charge the above mentioned flexible EV, represented by FO f through a flexible order. The duration of a flexible order is mapped to the number of slices of f , the volume to the power of the slices, and the time interval to the time flexibility of f . For

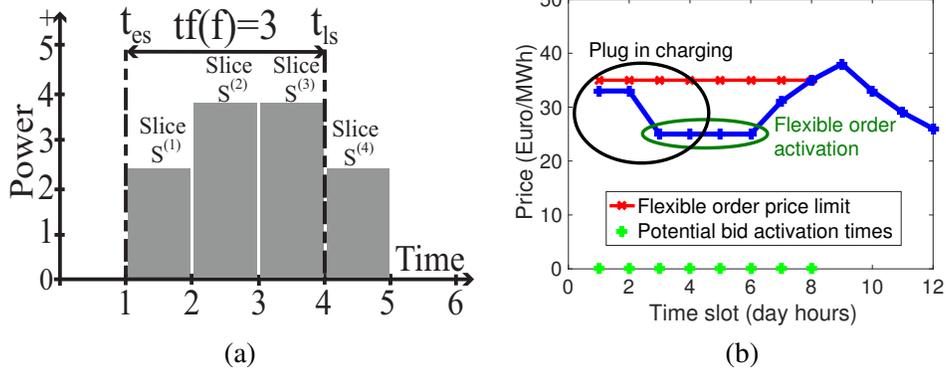


Figure 1: An example of an FO and a flexible purchase order

instance, a BRP could place a flexible order named “F1”, with duration 4 hours and time interval from 1 to 8. The volume of F1 is 0.0037MW (in order to satisfy all the slices) and its price limit is 35 euros/MWh. However, the energy needed to charge a single EV is too small to be traded in Elspot. In particular, the minimum contract size and the volume trade lot for a flexible order are both 100kW, while the power used by an EV is a few kW. Moreover, when the duration of a flexible order is more than one hour, the volume needed for these hours shall be constant. As a result, it is necessary to *aggregate* FOs to trade the flexible loads of the EVs through flexible orders in Elspot market.

The flexible order is activated in the time interval that optimizes social welfare provided that the price is respected [14]. Given F1 in a liquid market, the order is activated when the cost of buying the required energy is minimized. For instance, we see in Figure 1b that F1 is activated in time slots 3, 4, 5, and 6 where the price is 25 euros/MWh. Thus, the energy needed to charge the EV costs $25 \cdot 0.0037 \cdot 4 = 0.37$ euros. On the contrary, if time flexibility of the EV is disregarded, its charging occurs based on a price independent order and its plug-in time (time slot 1-4 in Figure 1b). In that case and according to Figure 1b, the cost is $33 \cdot 0.0037 \cdot 2 + 25 \cdot 0.0037 \cdot 2 = 0.4292$ euros, 16% more than the cost achieved by flexible order F1. The absolute difference (imbalance) between the purchased energy and the energy needed is traded in the balance market and usually for a higher price than the one in Elspot. Consequently, the BRP desires to be as precise as possible regarding the purchased energy from Elspot.

3 Problem Formulation

In this section, we show how aggregation of FOs that represent flexible charging loads of EVs can fulfill the requirements of flexible orders. We also introduce the problem of market-based aggregation.

3.1 FO aggregation

Based on [18], FO aggregation is the function that given a set of FOs F , produces a set of aggregated FOs AF where $|AF| \leq |F|$. Due to the time flexibility of the FOs, there are different alignment combinations that can lead to different AFOs. According to start-alignment FO aggregation, the earliest start charging time of an aggregated FO (AFO) f_a is the minimum earliest start charging time among all the FOs that produced it, i.e., $f_a.t_{es} = \min_{f \in F'}(f.t_{es}), F' \subseteq F$. The latest start charging time of f_a is the sum of its t_{es} and the minimum time flexibility among all the FOs in F' , i.e., $f_a.t_{ls} = f_a.t_{es} + \min_{f \in F'}(tf(f))$. The power profile of f_a is produced by summing up the power profiles of the FOs when they are aligned according to their earliest start charging time.

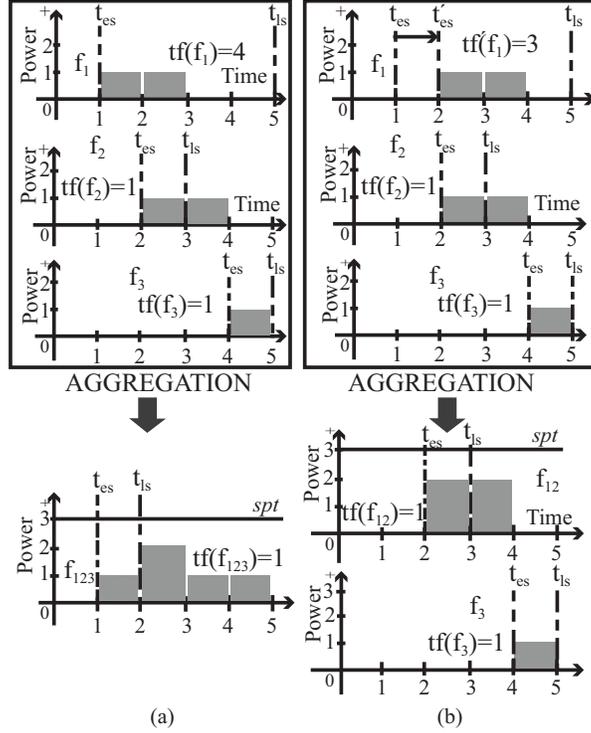


Figure 2: Flex-offer aggregation according to different alignments

For instance, we see in Figure 2a three FOs, $f_1 = ([1, 5], \langle 1, 1 \rangle)$, $f_2 = ([2, 3], \langle 1, 1 \rangle)$, and $f_3 = ([4, 5], \langle 1 \rangle)$, that produce AFO f_{123} where $f_{123}.t_{es} = f_1.t_{es} = 1$ and $f_{123}.t_{ls}$ is the sum of $f_{123}.t_{es}$ and time flexibility of f_2 or f_3 , i.e., $f_{123}.t_{ls} = 2$. The power profile of f_{123} is produced by summing up the power profiles of f_1 , f_2 , and f_3 based on their alignments. Thus, $f_{123}.s^{(1)}.p = f_1.s^{(1)}.p = 1$, $f_{123}.s^{(2)}.p = f_1.s^{(2)}.p + f_2.s^{(1)}.p = 2$, $f_{123}.s^{(3)}.p = f_2.s^{(2)}.p = 1$, and $f_{123}.s^{(4)}.p = f_3.s^{(1)}.p = 1$.

Due to the time flexibility of the FOs, there are different alignment combinations that can lead to different AFOs. For instance, given the 3 FOs f_1, f_2, f_3 in Figure 2 with time flexibility 4, 1, and 1, respectively, there are 20 ($5 \cdot 2 \cdot 2$) alignment combinations. As a result, based on different alignments, time flexibility of the FOs can be adjusted accordingly and different power profiles for the AFOs are produced. Moreover, a set of FOs can be partitioned and each subset can produce an AFO. Consequently, the output size of aggregation can be greater than one. For instance, we see in Figure 2b that the output of aggregation is 2 AFOs, i.e., f_{12} and f_3 . In particular, FO f_1 is aligned with f_2 and time flexibility of f_1 is adjusted so that $f_1.t'_{es}$ is equal to $f_2.t_{es}$. Consequently, the power profiles of f_1 and f_2 are summed up and they produce AFO f_{12} .

3.2 Market-based FO aggregation

Given a portfolio, the goal of a BRP is to maximize its profit by purchasing, for the minimum price, the energy that it sells to its customers. We consider flexible EVs to be part of a BRP's portfolio and, since the energy purchase takes place through orders, we examine if the energy needed to charge the flexible EVs can be purchased through flexible orders. The purchasing strategy of a BRP depends on many different factors, e.g., the content of the portfolio (factories, households, etc.) and pricing forecast. The strategy is out of scope of this work and left for Future work. However, since a flexible order has in general a higher probability to achieve a lower purchase price, we consider the goal of a BRP to be the maximization of the purchased energy through flexible orders.

In order for an FO to fulfill the flexible order requirements, the FO must have (1) time flexibility at least one and (2) between 1 and 23 slices. Moreover, since the minimum contract size and the trade lot of a flexible order are both 100kW, (3) the values of the slices of the FOs shall be multiples of 100kW. In our work, we introduce *market-based FO aggregation* (MAGG) to be the aggregation that given a set of FOs, outputs at least one AFO that fulfills the flexible order requirements. For illustrating purposes, we assume in our example below that *both the volume and the trade lot for a flexible order is 2kW instead of 100kW*. For instance, we see in Figure 2 that none of the individual FOs fulfills the power profile requirements of a flexible order (2kW). Thus, market-based FO aggregation is necessary. In that case, market-based FO aggregation produces AFO f_{12} that fulfills the flexible-order requirements since its time flexibility is 1 and the power of both the slices equals to 2, see Figure 2b. FO f_3 is also part of the aggregation output, but it is not a valid AFO because it does not fulfill the power profile requirement, i.e., its slice amount is lower than 2.

The flexible EVs are represented by a set of FOs. For instance, 5000 EVs that are part of a BRP's portfolio are represented by a set of FOs F . Each EV is an FO f of the set, i.e., $f \in F, f = (T(f), P(f)), T(f) = [t_{es}, t_{ls}], P(f) = \langle s^{(1)}, \dots, s^{(m)} \rangle$. A BRP must aggregate the FOs to produce AFOs that fulfill the flexible order requirements and can be then placed in the market as flexible orders. The volume of energy is expressed through the sum of the slices of the FOs and the power of each slice must be a multiple of 100kW. However, due to technical charging characteristics (EV power demand is in the interval [3.7kW,11kW] for household charging), we take into account a power range to define the valid power amounts. Thus, instead of considering exact multiples of 100kW for the power amount of each slice, we permit an insignificant amount deviation of e kW per slice, e.g., 5kW. When the financial evaluation of market-based aggregation occurs, the deviated amount will be considered to be traded in balance market, see Section 2.2. Hence, the problem of maximizing the bidden energy through flexible orders given a set of FOs is formulated as follows:

$$\begin{aligned}
& \text{Maximize} && \sum_{f_a \in A} \sum_{s \in P(f_a)} s \cdot p \\
& \text{subject to} && A = \text{MAGG}(F), 1 \leq |A| \leq 5 \\
& && \forall f_a \in A, t f(f_a) \geq 1 \\
& && \forall f_a \in A, 1 \leq |P(f_a)| \leq 23 \\
& && \forall f_a \in A, \forall s \in P(f_a), \\
& && s \cdot p = x \cdot 100\text{kW} \pm e\text{kW}, x \in \mathbb{N}_{>0}, e \in [0, 5]
\end{aligned}$$

Market-based FO aggregation complexity. Given a set of FOs F , the number of aggregation results that can be produced is: $\sum_{k=1}^5 \binom{|F|}{k} \cdot \text{avg}(al)$ where $\text{avg}(al)$ is the average number of alignments per partition [20]. For instance, given a set with 100 FOs and 20 alignments per partition on average, there are in total $1.3148 \cdot 10^{69}$ potential aggregation results (approximately the estimated number of atoms in the Milky Way Galaxy) that have to be examined in order to find the optimal one. Furthermore, tackling the problem as an Integer Linear Programming (ILP) problem, requires on the order of $\mathcal{O}(\text{avg}(al) \cdot |F| \cdot \sum_{k=1}^5 \binom{|F|}{k})$ decision variables to identify the partition(s) and the aggregation(s) that maximize the bidden energy. The complexity of the problem is thus too high to be solved by state-of-the-art solvers [21]. The reader can find a comprehensive version of the complexity analysis in Appendix A.

4 Heuristic solutions

Due to the unrealistically large solution space, we instead propose 3 variations of a heuristic algorithm, i.e., *Heuristic Market-based Aggregation Main Algorithm* (HMAMA) that tackles the market-based aggregation problem.

Algorithm 1 Heuristic Market-Based Aggregation

Input: F - set of FOs, e - amount deviation

Output: AF - set of AFOs

- 1: $continue \leftarrow true, AF \leftarrow \emptyset$
 - 2: **while** $continue = true$ **do**
 - 3: $ppt \leftarrow 23, spt \leftarrow 100$
 - 4: $PF, UF, f_{ini}, tft \leftarrow \text{Initialize}(F)$
 - 5: $PF, AF \leftarrow \text{Process}(PF, AF, f_{ini}, tft, ppt, spt, e)$
 - 6: $F, continue \leftarrow \text{Examine}(PF, UF, AF, continue)$
 - 7: **return** $\text{Top5EnergyAFOs}(AF)$
-

Algorithm 2 Longest Profile - Initialization phase

- 1: **function** $\text{Initialize}(F)$
 - 2: $f_{ini} \leftarrow \text{SelectAmongLongestTheMostFlexibleFO}(F)$
 - 3: **return** $F \setminus f_{ini}, \emptyset, f_{ini}, 1$
-

4.1 Heuristic Market-based Aggregation Main Algorithm

The goal of HMAMA is to produce AFOs that respect the flexible order requirements while avoiding the high complexity of the problem and at the same time provide good results in terms of bidden energy amount. Thus, given a set of FOs F , HMAMA (Algorithm 1) performs incremental binary aggregations so that the produced AFOs increase the captured energy in each step. In addition, the algorithm maps the flexible order requirements to threshold parameters that must be respected during the performed aggregations. Consequently, it introduces 3 thresholds, namely, the slice power (spt), time flexibility (tft), and power profile (ppt) thresholds that correspond to flexible order requirements. It sets spt to 100 since flexible orders must have multiples of 100kW power. Moreover, HMAMA assigns 1 and 23 to tft and ppt , respectively, since flexible orders must have a time interval of 1 and duration at most 23 hours. Permitted amount deviation is represented by e that is assigned values from 0kW to 5kW.

The body of HMAMA consists of 3 phases (functions), i.e., *initialization*, *processing*, and *examination* (Algorithm 1, Lines 2–6). During the initialization phase (Line 4), HMAMA identifies the FO with which to start binary aggregations (f_{ini}) and the subset of the FOs (PF) that participates in the aggregations. Then, during the processing phase (Line 5), it produces all the potential binary aggregations between f_{ini} and the FOs in PF to produce AFOs that fulfill the flexible order requirements. Afterwards, during the examination phase (Line 6), HMAMA examines whether it shall restart using the remaining FOs or terminate.

4.2 Main Algorithm variants

The initialization phase is salient for the outcome of the algorithm as it mainly defines the solution space that the algorithm explores. Hence, we introduce 3 variants of HMAMA that have different initialization phases, namely, the *Largest Profile* (LP), *Dynamic Profile* (DP), and *Dynamic Time Flexibility* (DTF).

LP focuses on producing AFOs with many slices because a long FO usually captures large energy amounts. On the other hand, given an FO with many slices, it is very difficult to fulfill the flexible order amount requirements and, especially, the slice amount equality required. For this reason, DP excludes from aggregation the FOs with extremely large profiles (outliers). DTF focuses on time flexibility of the FOs that has a prominent role in aggregation since it is directly correlated to the alignments. Thus, DTF takes into account the time flexibility distribution of the initial set and gradually excludes from aggregation the FOs with low time flexibility compared to the initial set.

Algorithm 3 Initialization phase - Dynamic Profile algorithm

```
1: function Initialize( $F$ )
2:    $uf \leftarrow$ UpperFenceProfileSize( $F$ )
3:    $PF \leftarrow$ FOsWithProfileAtLeastUF( $F, uf$ )
4:    $f_{ini} \leftarrow$ SelectTheMostFlexibleFOAmongLongest( $PF$ )
5:   return  $PF \setminus f_{ini}, F \setminus PF, f_{ini}, 1$ 
```

Algorithm 4 Initialization phase - Dynamic Time Flexibility

```
1: function Initialize( $F$ )
2:    $tft \leftarrow$ LowerFenceTimeFlexibility( $F$ )
3:    $PF \leftarrow$ FOsWithTimeFlexibilityAtLeasttft( $F, tft$ )
4:    $f_{ini} \leftarrow$ SelectTheMostFlexibleFOAmongLongest( $PF$ )
5:   return  $PF \setminus f_{ini}, F \setminus PF, f_{ini}, tft$ 
```

LP - Initialization phase. LP starts by selecting the most flexible FO among the ones with the largest profile size (Algorithm 2, Line 2). An FO with large profile size and high time flexibility has high probability to time-wise overlap with profiles of other FOs. So, AFOs that fulfill the flexible order requirements through different alignments can be produced. LP uses the initial set F as the processing set PF (Line 3) and then executes the processing and examination phase.

DP - Initialization phase. During the initialization phase, DP divides the initial set F into 2 subsets. First, DP computes the upper fence (uf) [22] of the power profile size of the FOs in F (Algorithm 3 Line 2). Then, it stores in PF the FOs that have profile size of at most uf (Line 3). It selects as f_{ini} the most flexible FO in PF among the ones with the longest profile and removes it from PF (Lines 4–5). For instance, given the set F in Figure 3a ($\{f_1, \dots, f_6\}$), uf is 4, see Figure 3b. DP excludes f_1 , which has a very long profile compared to the other FOs (red circle in Figure 3b), from F and selects FO f_6 as f_{ini} . FOs with very long profiles have difficulties satisfying the slice equality and it is likely that they have small time flexibility due to their long profiles (e.g., many charging hours for the EVs). Thus, they have less potential alignments to further satisfy the flexible order requirements. Then, DP continues aggregation with the processing and examination phase using PF , i.e., $F \setminus \{f_{ini} \cup f_1\}$.

DTF - Initialization phase. DTF takes into account the time flexibility distribution of the initial set F and excludes FOs with low time flexibility compared to the initial set. It computes the lower fence of time flexibility distribution of F and sets the time flexibility threshold (tft) equal to the lower fence [22] (Algorithm 4 Line 2). It splits F based on the lower fence of the time flexibility distribution in the set. It stores the FOs with time flexibility at least tft in PF (Line 3). DTF then selects f_{ini} from PF (Line 4). As a result, the algorithm excludes the FOs that have very small time flexibility. For instance, given the set F in Figure 3a, tft equals 6, Figure 3c. Thus, DTF excludes f_3 , which has very low time flexibility compared to the other FOs in the set, from F , see the blue circle in Figure 3c. DTF then sets tft to 6, selects FO f_1 as f_{ini} , and continues aggregation with PF , i.e., $F \setminus \{f_{ini} \cup f_3\}$. FOs with small time flexibility have lower probability to contribute in aggregation due to the low number of alignments that they have. Moreover, by setting tft equal to the lower fence, DTF reduces the number of examined alignments and consequently the complexity of the algorithm. Thus, AFOs with greater time flexibility are more likely to be produced.

Processing phase. In the processing phase, HMAMA examines all the potential binary aggregations between f_{ini} and the FOs in PF defined in the initialization phase. The FOs are examined in descending order according to their time flexibility. FOs with high time flexibility have more potential to participate in an aggregation that fulfills the flexible order requirements because of high number of alignments.

HMAMA examines, through the potential alignments, all the binary aggregations that fulfill the time

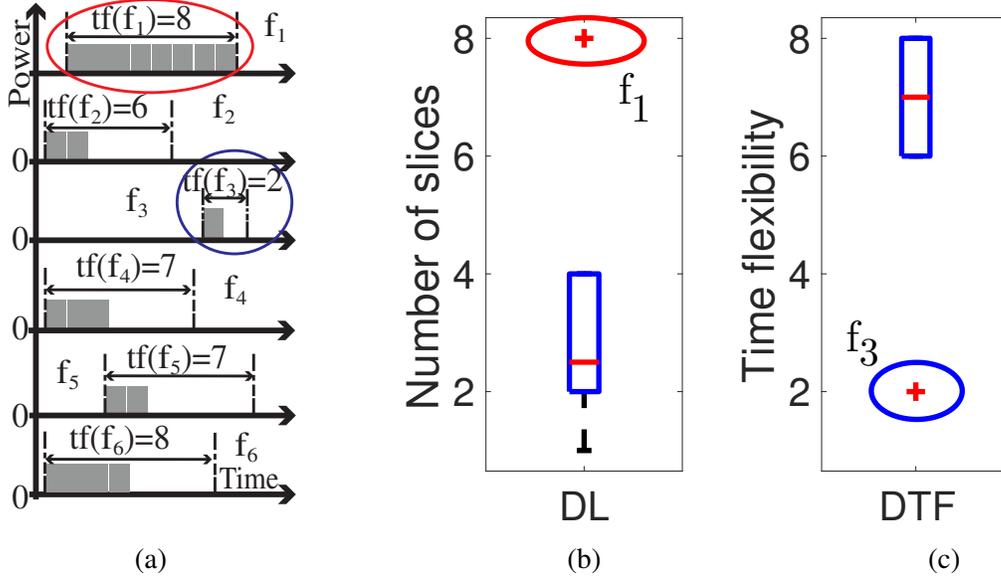


Figure 3: DL and DTF example, profile size and time flexibility box plots

flexibility tft and the power profile thresholds ppt (Algorithm 5, Lines 3–5). Among the AFOs that reduce the root mean square error (RMSE) between f_{ini} and the slice power threshold spt , it chooses the one with the minimum coefficient of variation (CV) (Lines 7–9). By promoting the reduction of RMSE, the produced AFO f_{cand} has a power profile closer to spt . In particular, the use of RMSE during aggregation prevents the increase of profile length of the potential AFO and contributes to the production of slices with values closer to spt . Consequently, alignments that lead to power profiles that time-wise overlap each other are preferred for aggregation. Moreover, because the slices of an AFO might have power deviations, the second condition of CV (Line 9) is used. A low CV of f_{cand} contributes to the elimination of power profile deviations and to the production of AFOs with slice power amounts closer to each other. For instance, given the FOs in Figure 2 and spt equal to 3, the RMSE between the slices of AFO f_{12} and spt is equal to 1 and lower than the RMSE between the longest FO f_{123} and spt , which is 1.8028. Similarly, f_{12} and f_{123} have CV equal to 0 and 0.4, respectively, with f_{12} having no power fluctuations. Thus, the reduction of RMSE and CV lead to AFOs that fulfill the flexible order energy requirements.

When an AFO with power amounts around spt is produced, an e kW deviation per slice is permitted (Algorithm 5 Line 12). At that point, an AFO f_a that fulfills the flexible order criteria is produced (Line 13). The FOs that participate in aggregation are temporally stored (Line 11) and when an AFO f_a is produced, they are removed from PF (Line 14). Then, spt is increased by 100 (Line 15) so that AFOs with larger energy are produced during the following aggregation. As a result, the processing phase produces an AFO that captures large amounts of energy and fulfills the time flexibility and power amount requirements of a flexible order. When all the FOs in PF are processed, HMAMA returns both PF and the output set AF with the aggregated FO f_a (Line 16).

Examination phase. During the examination phase, HMAMA first examines if there are any FOs in either PF or UF to further continue aggregation (Algorithm 6 Line 2). In case, the total energy of the remaining FOs is larger than the 5th in descending size energy AFO, HMAMA continues using the remaining FOs (Line 4). Otherwise, HMAMA does not continue the execution (Line 3). As a result, the algorithm ensures that the remaining FOs cannot produce an AFO with energy greater than one of the 5 produced AFOs. Since the 5 AFOs with the most energy will be transformed into flexible orders, the algorithm terminates (Algorithm 1 Line 7).

Algorithm 5 Processing phase

```
1: function Process( $PF, AF, f_{ini}, tft, ppt, spt, e$ )
2:    $PF_{tmp} \leftarrow \emptyset, f_a \leftarrow \text{null}$ 
3:   for each  $f \in PF$  do
4:      $f_{cand} \leftarrow \text{null}, bestCV \leftarrow \infty$ 
5:     for each alignment  $al$  of  $\{f_{ini}, f\}$  do
6:        $f_x \leftarrow \text{BinaryAggregation}(f_{ini}, f, al, tft, ppt)$ 
7:       if  $\text{RMSE}(f_x, spt) < \text{RMSE}(f_{ini}, spt)$  then
8:         if  $\text{CV}(f_x) < bestCV$  then
9:            $bestCV \leftarrow \text{CV}(f_x), f_{cand} \leftarrow f_x$ 
10:    if  $f_{cand} \neq \text{null}$  then
11:       $PF_{tmp} \leftarrow PF_{tmp} \cup f, f_{ini} \leftarrow f_{cand}$ 
12:    if  $\forall s \in P(f_{ini}), spt - e < s.p < spt + e$  then
13:       $f_a \leftarrow f_{ini}$ 
14:       $PF \leftarrow PF \setminus PF_{tmp}$ 
15:       $PF_{tmp} \leftarrow \emptyset, spt \leftarrow spt + 100$ 
16:  return  $PF, AF \cup f_a$ 
```

Algorithm 6 Examination phase

```
1: function Examine( $PF, UF, AF, continue$ )
2:   if  $PF \cup UF = \emptyset$  OR  $(|AF| \geq 5$  and
    $\text{totalEnergy}(PF \cup UF) < \text{Energy}5^{\text{th}}\text{AFO}(AF))$  then
3:      $continue \leftarrow \text{false}$ 
4:   return  $PF \cup UF, continue$ 
```

5 Experimental Evaluation

5.1 Experimental setup

We consider a BRP managing a portfolio of EVs represented by FOs. The BRP utilizes our proposed aggregation algorithms to produce AFOs that respect the flexible order requirements. The BRP transforms the 5 AFOs which capture the highest amount of energy into flexible orders and trades them in Elspot. In order to examine the scalability of our proposed algorithms, we create 8 differently-sized FO datasets, from 5K to 40K FOs (multiples of 5K), with characteristics based on the probability distributions suggested in [16]. Moreover, we consider that all EVs use the charging option described in Section 2.1 and need to be fully charged. Thus, the initial SOC of all EVs is within [20%, 85%], while they must be charged up to 90%. Details about the characteristics of the datasets are in Table 1.

We compare our techniques with two baseline aggregation techniques [18]. We use Start-Alignment (SA) aggregation, see Section 3.1 and Start-Alignment Grouping (SAG) aggregation. SAG groups together FOs that have both the same earliest start charging time and the same time flexibility and then applies SA on each group. As a result, it produces one AFO per group. We evaluate our techniques in terms of output size (#AFOs), participation of FOs in aggregation, percentage of energy traded in the market, running time, and both time flexibility and profile length of AFOs.

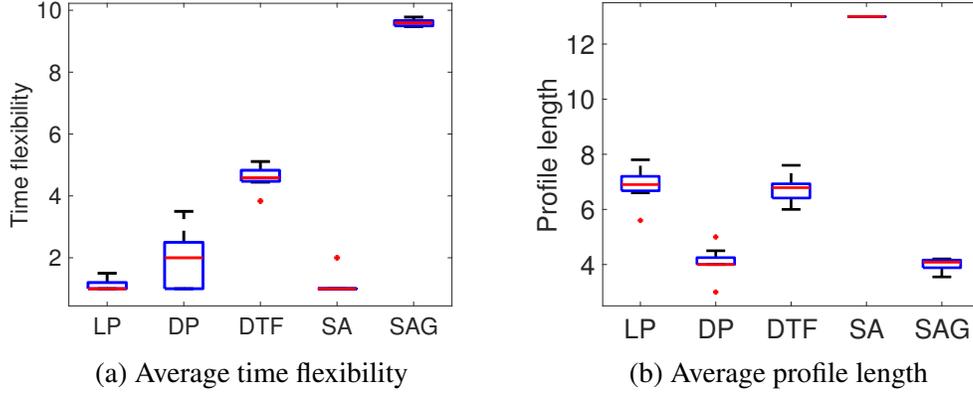


Figure 4: Average time flexibility, average profile length

	Distr.	Mean	St. dev	Min	Max
Battery capacity (kWh)	UD*	23	4	16	30
Arrival time	TGD*	19:00	2h	16:00	1:00
Departure time	TGD*	7:00	2h	5:00	12:00
Initial Battery SOE (%)	TGD*	75	25	20	85

* UD: uniform distribution, TGD: truncated Gaussian distribution

Table 1: EV data probability distribution

5.2 Market-based aggregation results

Output size. SA always produces one AFO whereas SAG produces more than 100 AFOs in all cases. Both LP and DP produce less than or equal to 5 AFOs in all cases. DTF produces more than 5 AFOs in 75% of the cases as the energy threshold is activated in a later step compared to the other techniques due to the division of the processed set.

Time flexibility and profile length. Regarding the baseline techniques, SA produces long AFOs with very low time flexibility as it aggregates all FOs into one. On the contrary, SAG produces short and time flexible AFOs due to the grouping phase it applies, see Figure 4a, b. LP uses as initial FO (f_{ini}) the longest FO of the dataset. Usually, such an FO has low time flexibility and so do the produced AFOs. Due to the long profile of f_{ini} , LP might utilize all the time flexibility of the remaining FOs to produce an AFO that reduces the distance to the power profile threshold (ppt). Consequently, LP produces long AFOs with very low time flexibility, see Figure 4a, b. The AFOs produced by DP are more flexible than the ones from LP since DP applies a dynamic profile size approach and excludes from aggregation very long FOs. As a result, FOs with similar profiles are aggregated together and less time flexibility is required to find a proper alignment that minimizes the distance to ppt . Consequently, AFOs with less slices compared to LP are produced, see Figure 4b. Finally, DTF produces the most flexible AFOs among our proposed techniques. We see in Figure 4a that the average time flexibility of the produced AFOs is greater than 4 in all datasets. DTF achieves it by utilizing the time flexibility threshold. However, DTF produces long AFOs, similar to LP, because it also selects as f_{ini} the longest AFO of the processed set, see Figure 4b.

Participation and traded energy. In order to quantify the participation of FOs in aggregation, we take into account only the FOs that participate in the aggregation of the 5 (or less) largest in energy AFOs, i.e., the AFOs that are transformed into flexible orders. Similarly, we compute the traded energy by taking into account only the energy captured by the AFOs that are transformed into flexible orders.

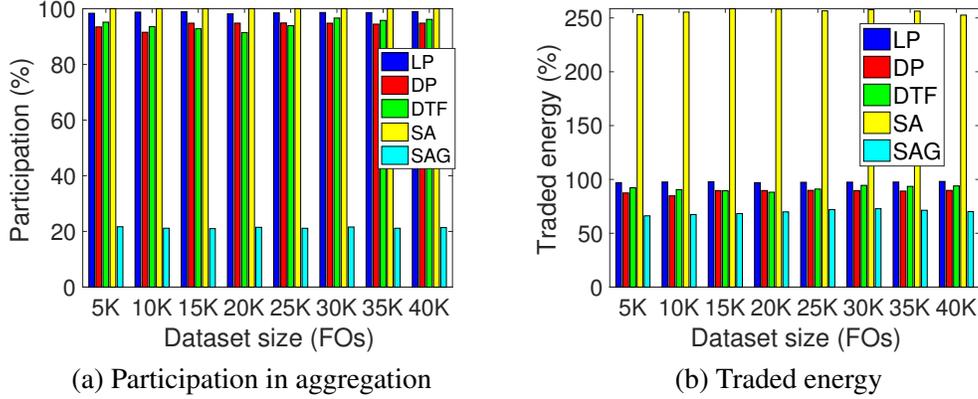


Figure 5: Participation of FOs and traded energy

SA aggregates all FOs into one AFO and thus participation in aggregation is 100%, see Figure 5a. The slices of the AFO have very high power differences and since a flexible order requires a flat power profile, the power of the highest slice is considered for the whole profile of the AFO. As a result, on average, 2.5 times the energy captured by that AFO is traded, see Figure 5b. On the contrary, SAG produces too many AFOs and since only the 5 largest are traded, we see a very low participation percentage and the lowest percentage of traded energy among the techniques (69.7% on average). In general, the longest AFOs capture more energy as they have more slices and more FOs participate in their aggregation. Thus, LP, which produces the longest AFOs, obtains both the highest participation percentage (98.6%) and traded energy percentage (97.5%) in all the cases among our proposed techniques, see in Figure 5a, b. DTF follows with an average participation value of 94.4% and 91.7% percentage of traded energy. DP has the lowest percentage in both participation and traded energy, 94.2% and 88.8% on average respectively. The reason is that DP excludes very long FOs, which usually capture large energy, from aggregation.

Processing time. Both SA and SAG are fast techniques with processing times below one second as they examine a very small solution space and do not consider the market requirements. LP is the fastest among all our proposed techniques since it efficiently activates the energy threshold, see Figure 6a. The processing time of DP follows a close to linear growth rate. DTF has an increasing trend for processing time, but it shows similar processing times for datasets with different sizes, e.g., for datasets with 30K and 35K FOs. The reason is that the processing time is highly driven by the number of initialization phases. The size of the dataset might increase, but the new added FOs might lead to less initialization phases and therefore to less aggregation comparisons. That is why we also notice that both processing time and number of initialization phases follow similar patterns. Whenever the number of initialization phases is increased compared to the previous dataset, processing time also increases. For instance, we see in Figure 6b that when the size of the dataset is increased from 15K to 20K for both LP and DTF, the number of initialization phases is reduced. As a result, the processing time is similar for both the datasets and slightly increases for the 25K. Eventually, when the size of the dataset is further increased, it becomes more difficult for DTF to fulfill the market requirements and thus both the initialization phases and the processing time are highly increased.

5.3 Financial evaluation

Since the overall goal of a BRP is to trade the AFOs in the market using flexible orders, we financially evaluate our aggregation techniques. We compare the cost of buying the energy needed to charge the EVs

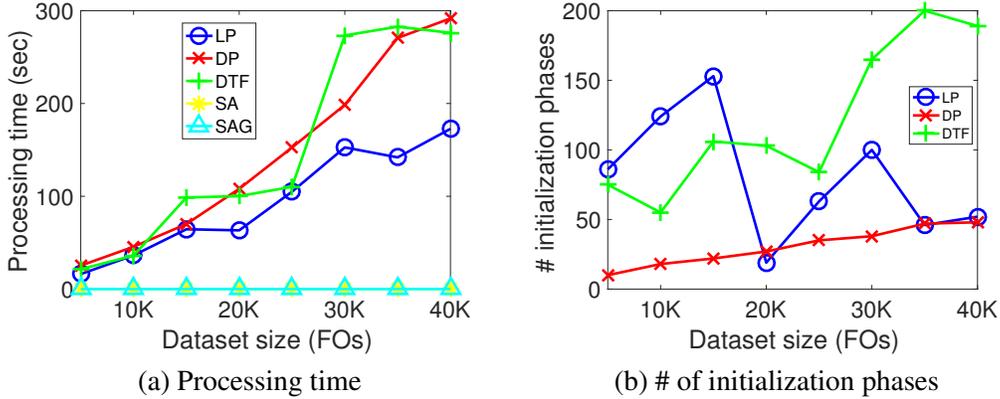


Figure 6: Processing time and number of initialization phases

based on plug-in time (traditional approach) with the cost of charging the EVs by utilizing flexible orders. Moreover, in order to compare our techniques with the optimal solution, we consider a *non-realizable in practice* scenario where each FO directly participates in the market without aggregation and each EV is charged when the charging cost is minimized.

Due to the fact that flexibility appears during the night [11], we consider a 48 hours trading period with a repetition of the 24h Elspot average prices of 2016 [14], see price curve in Figure 8a. In the same figure, we illustrate the time and the energy amount used to charge the 40K dataset based on our techniques, the two baseline techniques, the plug-in times of the EVs, and the optimal charging. We see that the charging of the EVs based on the plug-in time occurs when the prices are still high and it does not take advantage of the price drop that occurs in the night of the first 24 hours.

SA and SAG produce AFOs that do not fulfill the market requirements. As a result, more energy than it is needed has to be traded in the market. In particular, SA trades 1.52 times more energy than needed to charge the EVs. Thus, the surplus energy is traded in the regulation market and it results in losses for the BRP, see negative cost reduction in Figure 8b. Regarding SAG, the produced AFOs capture a low percentage of the energy needed and they also require extra energy to be traded in order to fulfill the market requirements. Consequently, the cost reduction due to the flexible orders trading is compensated by the losses from the surplus energy trading. As a result, we see only 1.1% cost reduction on average when SAG is applied. On the contrary, the optimal charging option charges all the EVs when the price has the lowest value. That is why we see a spike in the graph reaching 180MW after the 24th hour.

Our proposed aggregation techniques also take advantage of the lowest prices. LP produces long AFOs which expand over many hours and have low time flexibility. That is why we see in Figure 8a that part of the charging occurs when the prices are high. DTF produces AFOs that are also long, but they are more flexible than the AFOs produced by LP. Therefore, EVs are charged when prices are a bit lower and DTF achieves a higher cost reduction, Figure 8b. Finally, DP produces short and flexible AFOs. As a result, it

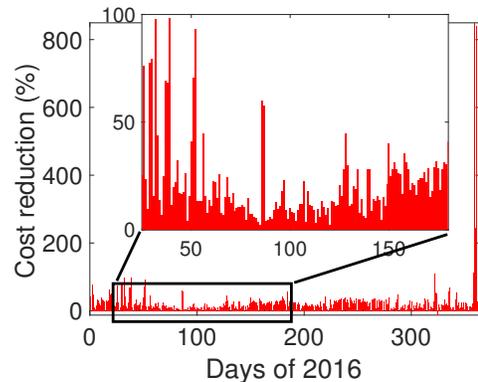


Figure 7: Cost reduction based on DP

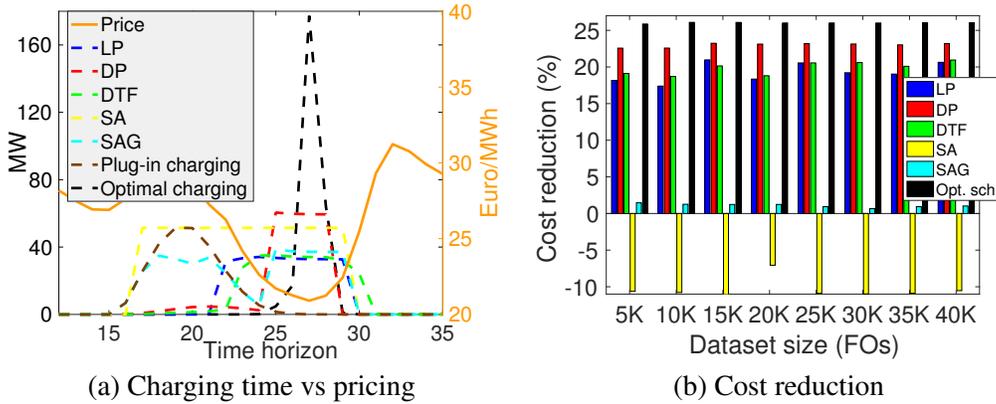


Figure 8: Charging times and pricing for 40K dataset and cost reduction for all datasets

takes advantage of the lowest prices occurring only for a few hours, see Figure 8a.

When the energy for the 40K FOs dataset is purchased based on the plug-in times of the EVs, it costs 7,521 euros. On the contrary, when LP, with the highest participation, is applied on the 40K dataset, 39,584 FOs participate in aggregation, see first bar (98.96%) in Figure 5a. The 39,584 FOs produce 5 AFOs which are further transformed into flexible orders. The cost of purchasing the energy needed for the 5 AFOs is computed based on the flexible orders trading and it is 5,851 euros, see Figure 8b. The price also includes the cost (0.40 euro) of the imbalances (62kW) of the flexible orders, see Section 2.2. The energy needed for the remaining 416 (40,000 – 39,584) FOs is bought based on their plug-in time and it is 117 euros. Thus, the overall energy bought to charge 40K EVs, when LP is used, costs $5,851 + 117 = 5,968$ euros. Therefore, LP achieves a 20.65% cost reduction in energy purchase, see LP bar for 40K dataset in Figure 8b.

We see in Figure 8b that DP achieves on average a 23.01% cost reduction. DTF follows with 19.87% and LP with 19.29% average cost reduction. The cost reduction based on the optimal solution is 26.01% on average. Thus, LP, DTF, and DP achieve 74.2%, 76.4%, and 88.5% of the optimal cost reduction, respectively. Notably, the cost reduction that DP achieves *only* for the FOs that participate in aggregation is on average 96.6% of the optimal one.

In Figure 7, we illustrate the cost reduction that DP achieves during 2016 (leap year). We consider 365 trading periods of 48 hours. The first trading period includes both the 1st and the 2nd day of 2016. The second trading period includes the 2nd and the 3rd day of 2016 and so on. The average cost reduction is 27.57% and, interestingly, we notice at the end of the year a cost reduction of more than 800%. The reason is that for several consecutive days, Elspot prices were negative early in the morning and even reached -53 euros/MWh on the 27th of December at 3:00.

Summary: By applying our proposed techniques on the aforementioned 365 trading periods, DP achieves the highest cost reduction in 52% of the periods and DTF achieves the highest cost reduction in the remaining 48% of the periods. The reason is that the financial impact of the techniques is highly correlated with the pricing curve of the trading period. Thus, in cases where the price drops for only few hours close to the plug-in charging time, DP is the most suitable technique. On the other hand, when the price drops for longer periods but much later than the plug-in charging time, DTF achieves a higher reduction than DP.

6 Conclusion and Future work

This paper investigates the market-based aggregation problem using the FO model that captures flexible charging loads of EVs. It proposes 3 market-based FO aggregation techniques that efficiently aggregate loads from thousand of EVs taking into account real market requirements. Consequently, the techniques produce aggregated FOs that can be transformed into flexible orders and be traded in the energy market. The paper financially evaluates the proposed techniques based on real electricity prices and shows that a 27% cost reduction on energy purchase can be achieved via flexible orders.

In our future work, we will enrich our techniques considering pricing forecast models and uncertainty in patterns of driving behavior. Moreover, we will examine a price-maker market scenario and different market strategies for the BRPs.

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A Market-based Flex-Offer aggregation complexity

A.1 Number of solutions

Given a set of FOs F , there are $\left\{\begin{smallmatrix} |F| \\ k \end{smallmatrix}\right\}$ ways (Stirling numbers of the second kind [20]) to partition the $|F|$ FOs into k subsets. Applying aggregation on each subset produces an AFO. In market-based FO aggregation, the size of the output is between 1 and 5. Thus, k can be assigned values from 1 to 5. Therefore, there are $\left\{\begin{smallmatrix} |F| \\ 1 \end{smallmatrix}\right\}$ ways to partition $|F|$ FOs into 1 non-empty subset of FOs. There are $\left\{\begin{smallmatrix} |F| \\ 2 \end{smallmatrix}\right\}$ ways to partition the $|F|$ FOs into 2 non-empty subsets, where the aggregated FOs are 2 and so on, given $|F|$ FOs, there are $\left\{\begin{smallmatrix} |F| \\ 1 \end{smallmatrix}\right\} + \left\{\begin{smallmatrix} |F| \\ 2 \end{smallmatrix}\right\} + \dots + \left\{\begin{smallmatrix} |F| \\ 5 \end{smallmatrix}\right\} = \sum_{k=1}^5 \left\{\begin{smallmatrix} |F| \\ k \end{smallmatrix}\right\}$ ways to partition the FOs.

Moreover, the number of the different aggregated FOs depends on the alignments of the FOs that participate in aggregation and thus on their time flexibility. In particular, given a set of FOs SF ($SF \subseteq F$) with time flexibility $tf(f_1), \dots, tf(f_{|SF|})$ respectively, the number of the aggregation results (aggregated FOs) that can be produced is: $\prod_{i=1}^{|SF|} tf(f_i)$.

Example 1. Given a set with 100 FOs there are $\sum_{k=1}^5 \left\{\begin{smallmatrix} 100 \\ k \end{smallmatrix}\right\} = 6.5738 \cdot 10^{67}$ potential partitions that can produce from 1 to 5 AFOs. Assuming a lower bound of 4 alignments per partition on average, there are in total $4 \cdot 6.5738 \cdot 10^{67} = 2.62952 \cdot 10^{68}$ potential aggregation results that have to be examined in order to find the optimal one.

A.2 Integer Linear Programming problem complexity

Given a set of FOs F , there are $\left\{\begin{smallmatrix} |F| \\ k \end{smallmatrix}\right\}$ ways (Stirling numbers of the second kind [20]) to partition the $|F|$ FOs into k subsets. Applying aggregation on each subset produces an AFO. In market-based FO aggregation, the size of the output is between 1 and 5. Thus, k can be assigned values from 1 to 5. Therefore, there are $\left\{\begin{smallmatrix} |F| \\ 1 \end{smallmatrix}\right\}$ ways to partition $|F|$ FOs into 1 non-empty subset of FOs. There are $\left\{\begin{smallmatrix} |F| \\ 2 \end{smallmatrix}\right\}$ ways to partition the $|F|$ FOs into 2 non-empty subsets, where the aggregated FOs are 2 and so on... Given $|F|$ FOs, there are $\left\{\begin{smallmatrix} |F| \\ 1 \end{smallmatrix}\right\} + \left\{\begin{smallmatrix} |F| \\ 2 \end{smallmatrix}\right\} + \dots + \left\{\begin{smallmatrix} |F| \\ 5 \end{smallmatrix}\right\} = \sum_{k=1}^5 \left\{\begin{smallmatrix} |F| \\ k \end{smallmatrix}\right\}$ ways to partition the FOs. As a result, tackling the market-based aggregation problem as an Integer Linear Programming (ILP) problem, requires on the order of $\mathcal{O}(|F| \times \sum_{k=1}^5 \left\{\begin{smallmatrix} |F| \\ k \end{smallmatrix}\right\})$ decision variables to identify the partition(s) that maximizes the bidden energy.

Moreover, the number of the different aggregated FOs depends on the alignments of the FOs and thus on their time flexibility. Therefore, for each FO, all potential start charging times have to be examined and that requires a number of decision variables equal to the time flexibility. Hence, given an average time flexibility \overline{tf} of set F , the number of the decision variables needed to identify the partitions and the aggregations that maximize the bidden energy is on the order of $\mathcal{O}(\overline{tf} \times |F| \times \sum_{k=1}^5 \left\{\begin{smallmatrix} |F| \\ k \end{smallmatrix}\right\})$.

For instance, given a set with 100 FOs there are $\sum_{k=1}^5 \left\{\begin{smallmatrix} 100 \\ k \end{smallmatrix}\right\} = 6.5738 \cdot 10^{67}$ potential partitions that can produce from 1 to 5 AFOs. Thus, $100 \cdot 6.5738 \cdot 10^{67}$ variables are needed to identify the proper subset(s) for all the FOs. Moreover, given an average time flexibility of 5, $5 \cdot 100 \cdot 6.5738 \cdot 10^{67} = 3.2869 \cdot 10^{70}$ variables are needed to identify the aggregation result that maximizes the bidden energy.